1. This is a time-homogeneous Markov chain because we assume that are all constant through time. It is a Markov chain because in order to understand the current state of the system all you need is its state one time step back.
2. This is the correct transition matrix because for a given previous time step the resulting value when passed through this matrix is the probability of the first player playing a different opponent. Additionally the rows need to add up to 1 and they do because each one contains both the probability of two players playing and the probability of them not playing, which adds up to 1.
3. Using wolfram alpha we get that . Now we need to divide by the sum of this vector so it adds up to 1. . Now we need to multiply this into the matrix to show its the correct equilibrium distribution.
4. This distribution makes sense as even though it may appear like X and Y with be playing 49% of the time, and X and Z will be playing 30% of the time, in reality all players will get a good amount of play time given this distribution as two players can’t play each other consecutively.
5. If they are all equally good then the equilibrium distribution should be and given they are all evenly good so we can verify our equilibrium distribution through eigen-analysis. . The analysis confirms this is the correct distribution.